Physically Based Smoke Animation

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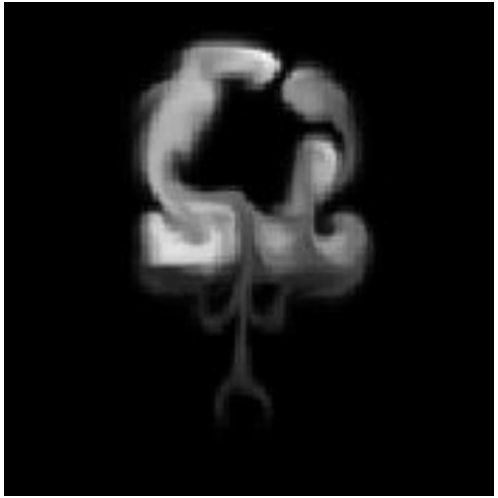
Keyframe Animation



$\rho \longrightarrow \rho^*$

Target Driven Smoke Animation (Fattal & Lischinski 2004)

Figure 4: Fluid logo. The five targets are shown in the top row, and six frames from the resulting animation are shown in the bottom row.

Smoke Animation

128 spp
128 x 128
30 minutes
140 its/s

Smoke Animation

128 spp
256 x 256
7 hours
50 its/s

Euler's Equations for Inviscid Flow

Density: $\rho(\mathbf{x}, t)$

Velocity: $\mathbf{u}(\mathbf{x}, t)$

Balance of momentum $\mathbf{u}_t = -\mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \mathbf{F}(\rho, \rho^*) - \mathbf{u}$

Incompressibility $\nabla \cdot \mathbf{u} = 0$

Advection: $\rho_t = -\mathbf{u} \cdot \nabla \rho$

Smoke Simulation

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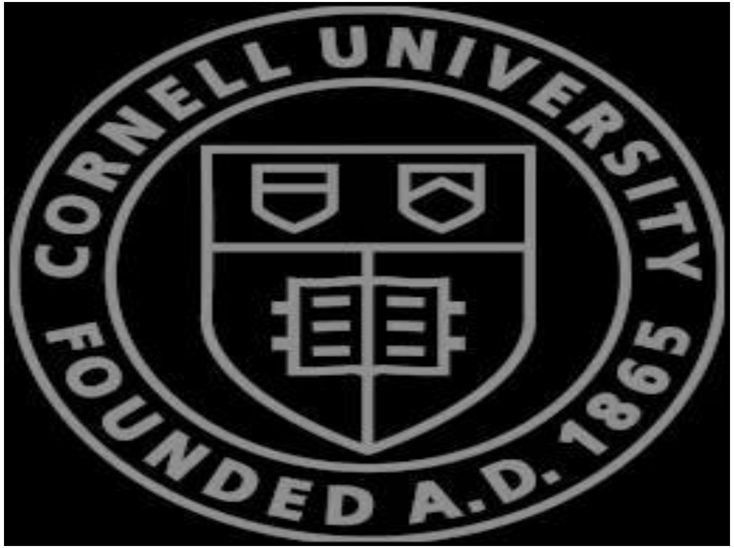
1. Apply driving forces $\mathbf{u}_t = v_f \mathbf{F}(\rho, \rho^*)$
2. Attenuate momentum $\mathbf{u}_t = -v_d \mathbf{u}$
3. Advect momentum $\mathbf{u}_t = \mathbf{u} \cdot \nabla \mathbf{u}$
4. Project: solve for pressure field given Poisson equation, and ensure zero divergence $\nabla^2 p = \nabla \cdot \mathbf{u}$
5. Advect smoke $\rho_t = -\mathbf{u} \cdot \nabla \rho$
6. Gather smoke $\rho_t = v_g \mathbf{G}(\rho, \rho^*)$

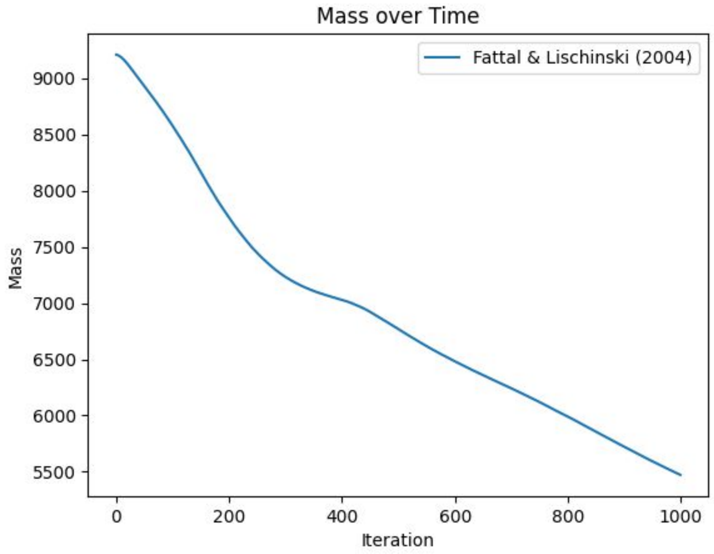
Neumann Boundary Conditions

2D Smoke Simulation



Limitations

1. No discretization invariance
2. No conservation of mass



What if we could parameterize the field continuously?

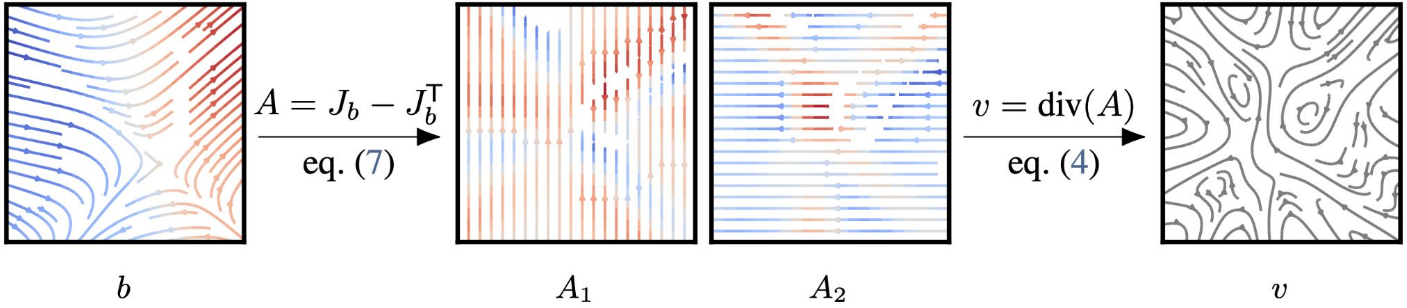


Figure 1: Divergence-free vector fields $v : \mathbb{R}^d \to \mathbb{R}^d$ can be constructed from an antisymmetric matrix field $A : \mathbb{R}^d \to \mathbb{R}^{d \times d}$ or an arbitrary vector field $b : \mathbb{R}^d \to \mathbb{R}^d$. J_b represents the Jacobian matrix of b, and A_1 and A_2 are the first and second rows of A. Color denotes divergence.

Neural Conservation Laws (Ritcher-Powell, Lipman, & Chen 2022)

How does Neural Conservation Laws (NCL) work?

Represent the domain as a **k-form field**. Use automatic differentiation to efficiently store Jacobians

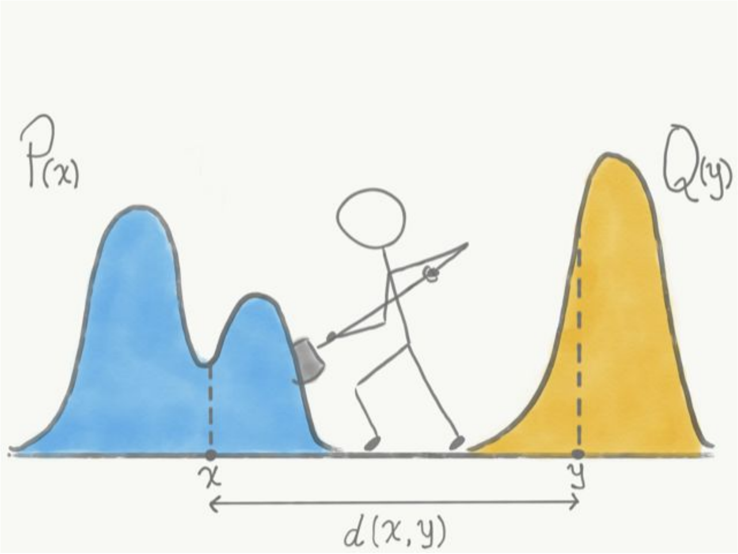
Parameterize the density and vector field via the **continuity equation**

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho u) = \mathrm{div}\begin{pmatrix} \rho \\ \rho u \end{pmatrix} = 0$$

Dynamical Optimal Transport: A Simulation Free Method

- Find a vector field between two distributions
- Mass must not “teleport”
- Transport must be smooth

$$\min_{\rho,u} \int_0^1 \int_\Omega \|u(x,t)\|^2 \rho(x,t)\,dx\,dt$$



Strategy

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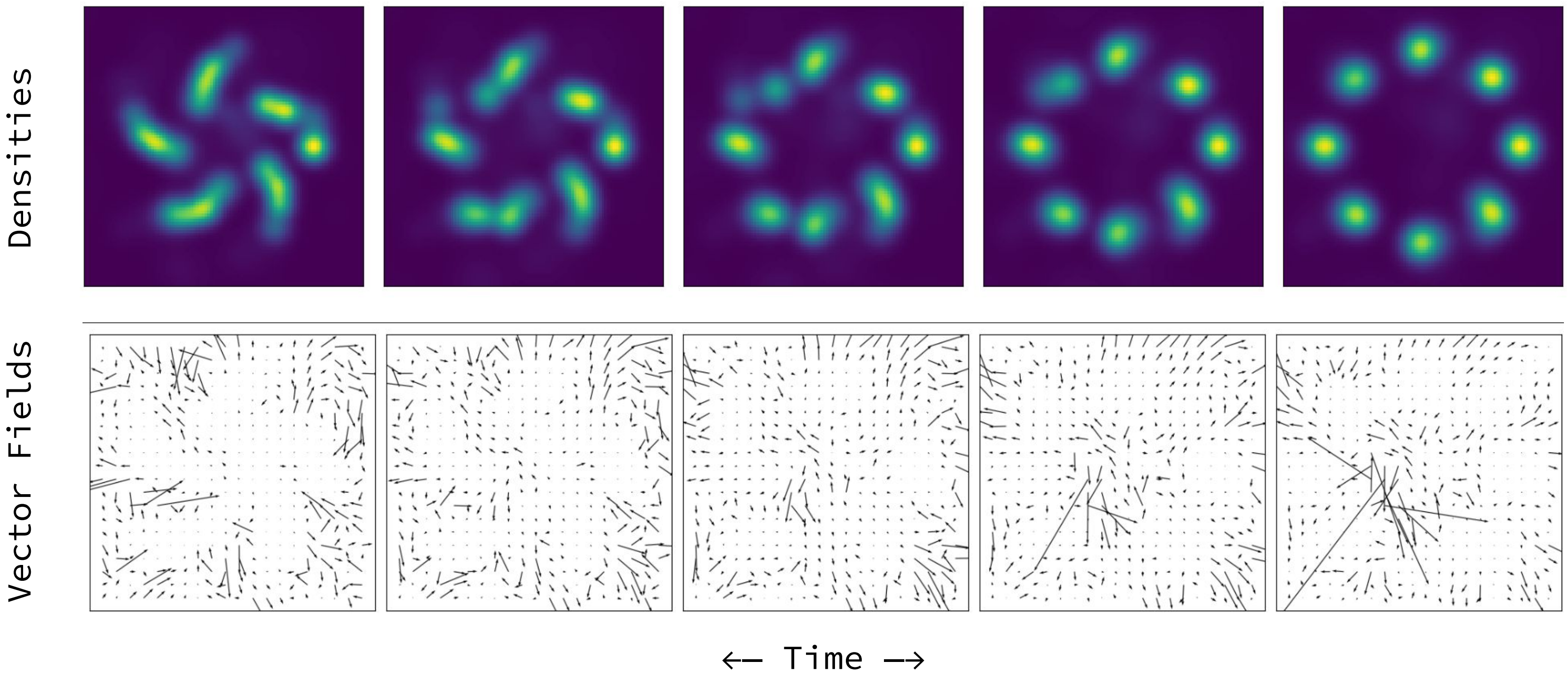
1. Learn an optimal transport map from one density to another
2. Ensure conservation of mass via the continuity equation
3. Ensure that vector field is divergence free
4. Ensure balance of momentum

Momentum Conservation Penalty

$$L_F = \|\mathbf{u}_t + (\nabla \cdot \mathbf{u})\mathbf{u} + \frac{\nabla p}{\rho}\|_\Omega^2$$

Results



Rendering

Summary of What I Did

- Implemented the Fattal & Lischinki 2004 paper in Taichi
- Integrated Taichi with Mitsuba to volume render the smoke plumes
- Reframed smoke animation as an optimal transport problem.
  - Attempted to add penalty terms to the OT model to model pressure.

Future Work

- Better model pressure and vorticity in dynamical optimal transport
- Explore more discretization free methods in graphics
- Explore the use of functional programming and differential forms in computer graphics

Grading Notes from Steve

This is a nicely defined and mostly well executed project. You have a good basic 2D fluid simulator running in Taichi, and on top of that you have implemented the Fattal & Lischinski target driven animation method. The results appear like reasonable fluid flows, though generally it doesn't seem to gather into the final shape as effectively as it should. It would have been interesting to explore the effects of the different parts of the solver a bit to better understand why this might be the case. But this is not a particularly simple little simulator to get working, particularly as a solo project!

The extension with fluid flow by optimal transport is interesting; I don't fully understand the formulation from the slides, and there's no report, so I'm not entirely sure what to make of it, or even what is new and what already existed. But it is great that you are exploring in a completely self-directed way!

Score; 3.9 for smoke control sim that almost works; +.2 for the extra piece = 4.1/4.0